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THE 6th Journal

Full reports of Action Research Projects

Foreword:

The 2005/6 academic year was a highly successful one for practitioner research in the college. Five member of staff carried out projects in diverse areas, and each of these have sparked interest and curiosity among other colleagues about strategies which can be used to improve learner's motivation and to engage them in the learning process. These projects have been written up in a format which seeks to ground the ideas in some background theory but also in a personalised way which hopefully other teachers will find accessible and will be able to relate to their own practice. If, having read the reports, you are inspired to find out more, I am sure that the relevant members of staff would be happy to speak about their projects. Alternatively, you may have an idea for a project yourself, which you can talk to me about.

David Godfrey
Senior Project Leader

Improving student participation and understanding in Mathematics through increased use of student problem setting and solving.

Kate Halpin

INTRODUCTION

The aims are:

1. To determine whether requiring students to set and solve their own mathematical problems for a fellow student to solve improves their participation in and enjoyment of lessons.
2. To determine whether students gain a clearer understanding of the underlying mathematics when they are required to set problems.
3. To determine whether students acquire a better understanding of the use of mathematical language through problem setting.

Background:

In September 2004 I started teaching a group of Single Maths students who had chosen Decision Maths as their applications module. They were a group that immediately jelled and obviously enjoyed working together. When I started the Decision 1 module I tried an idea that a colleague had suggested to me which was to set students the task of making up their own questions and presenting them to a fellow student to solve.

I first tried this with the topic on Dijkstra's algorithm for finding the shortest path through a network. I was surprised at how enthusiastically and imaginatively the students went about the task. They enjoyed the activity very much and I believe they understood that topic well thereafter.

I used this idea in a fairly haphazard way from then on, but was always impressed at how well these activities were received and executed.
(That particular group went on to do well in their AS Maths and in the second Decision module. Their teachers of the second module taught it in a similar way and probably more imaginatively than I did. It has been very gratifying to see such success and these students have been very active mathematically since.)

I could not be sure whether it was just a property of the group that they did well in the end and that they enjoyed question setting exercises, but I was determined that I would continue to incorporate question setting into my lessons. I also wanted to be sure that, even though I am nearing the end of my teaching career, I continue to explore teaching ideas with which I am less familiar. Further, at our college we are encouraged to reflect on recent research into learning styles to enhance our teaching and our students' learning.

In September 2005 I decided that I would make question setting a regular activity for the lower 6 Double Maths group I share with a colleague. I felt there would be many topics for which I could confidently try this method of teaching and learning. I told the students I would be doing this research and their response was immediately cooperative and interested. It seemed to me that they liked the idea of being part of some research, but I believe they also liked the idea of a slightly different type of activity.

Theory

Early in my teaching career I used overwhelmingly a deductive teaching style. This involves beginning with general principles and deducing methods and applications from them. This leads to concise and highly structured presentations. By contrast, inductive teaching allows students to see specific cases first and then work up to the principles involved by inference via lessons that may have less rigid structure. Felder (1993) points out that research has shown that "of these two approaches to education, induction promotes deeper learning and longer retention of information and gives students greater confidence in their problem-solving abilities". This struck me as an approach to teaching that I should use more consistently. I believe that when students set their own questions they will need to think about the underlying ideas, solve their own problem and sort out any mistakes they might make, including those made by setting awkward or unworkable problems. I believe this is an aspect of inductive learning that can be used in conjunction with others.

I was also impressed by a paper written earlier by Felder and Silverman (1988) in which they reported that "Much research supports the notion that the inductive teaching approach promotes effective learning. The benefits claimed for this approach include increased academic achievement and enhanced abstract reasoning". This also gave me the impetus to try this method systematically as one of my aims was to enable my students to gain a clearer understanding of the mathematics they are studying.

Mathematics is a discipline that requires reasoning and thinking which may not be engaged in during the normal daily course of events. Honing the appropriate thinking skills for the subject should surely improve enjoyment, understanding and facility in it. There is research being done in the post-16 sector on "thinking skills" approaches to teaching. Findings have shown this to be effective in improving ability.

In particular using "peer-interaction" to aid development of thinking skills via "collaborative researching, thinking and discussing together" has been found to be effective. Moseley, Baumfield et al (2004). I believe the method I am using will enable students to develop thinking skills. I also hope that they will enjoy the process as most of my students in this group have declared that their preference is for interpersonal, as opposed to intrapersonal, learning.

I was mindful that teaching by requiring students to exchange questions they had set would not suit every student's learning style. However, as stated by Atherton "pandering to learning styles may be doing the students a disservice: they will benefit more from adapting and becoming versatile, more able to respond both to formal teaching and learning from experience, than they will from having everything made as easy as possible for them in your particular subject". Atherton J.S. (2002). I therefore felt little anxiety at proceeding in the hope that students averse to this method might become more flexible in their learning styles.

METHOD

For each of the activities a clear description of the task was given a day or two in advance, for homework. Students were to set a question on the given topic, solve it carefully, respecting the notation and language of Maths to make sure it was "solvable" with the methods and techniques they were learning. I was particularly keen for the students to engage in the thinking behind problem solving.

In class the next day (or 2 later) they would be matched randomly with another student (I use my own, handmade cards to do this). Pairs would then swap their prepared questions and solve them. The pairs would then check each other's solutions and comment on them.

I hoped this method will appeal to introvert and extrovert students alike. Introvert – as they would have the opportunity to reflect via their preparation, extrovert – as they would have the opportunity to share problems in pairs (possibly a larger group too).

FINDINGS

I shall now record some of the question setting activities done throughout the year. I have recorded the first in some detail as it is a very general type of problem that may be used in other disciplines. Thereafter I shall record the tasks briefly only. Further details may be found in the appendix. Students' questions and some of their work are in the appendix.

Task 1: Algorithms – 13th September 2005
Students were set the task to make up an algorithm. The idea was that the algorithm should result in an outcome which would not initially be known to the student who was to apply it. This idea was "borrowed" from a colleague who had tried it out with his own students when teaching them Decision 1 and found it highly workable.
The task was set a day in advance of the lesson. Examples of some of the algorithms that the students came up with were:

- Making a paper aeroplane. This was successfully made by the student pair.
- Drawing a simple but abstract shape. Successfully done with pencil.
- An algorithm that calculates the longest chain of even numbers in a given series of integer numbers – probably off the internet. A good challenge.
- Directions to a student's home.
- Finding out whether a number is prime. A difficult challenge.

This turned out to be a highly successful exercise to start this research because it is fun, not difficult to understand mathematically and could be made as challenging as the student wished, and, indeed, a few students really rose to the challenge of finding or devising tricky algorithms.

During the activity I did not intervene at all. This was not necessary as the students were trying each other's problems with much animation and the level of discussion was much higher than on occasions when they would engage in other types of group or paired activities. I was extremely pleased that they were quite critical about the algorithms set and the steps they used. I was particularly pleased that students did not appear to be unhappy about criticism. In fact this group has since worked very well together in a well-balanced and productive way. This is also due, to a large extent, to their other mathematics teacher using very innovative teaching methods.

Task 2: Graph theory and networks – 20th September 2005
Students were set the task to make up a question about graphs and networks. They had just spent the lesson researching graphs and network via a verbal and visual group exercise and the aim of this activity was to consolidate their learning and to differentiate between deeper and superficial understanding. Students were told they would swap their questions with another student, at random, the next day.

I was pleased with the outcomes of this task because it showed the students the need to be precise in giving definitions and descriptions in mathematics. Ambiguity in mathematics is undesirable (at least at this level). Some students found this task a bit more difficult and one was rather crestfallen when what he had produced did not work very well. I thought this was not at all a bad thing as it stressed the need for precise definitions. However, I felt that this activity had not been as positively embraced by all the students as the previous had been.

QUESTIONNAIRE 1

At this stage I wanted to gauge formally how students regarded this method of setting and solving problems. I did this using a questionnaire.

Homework Styles Questionnaire – 12th October 2005

	V+	+	N	-	V-
1. I enjoy homeworks for which I have to prepare questions for a fellow student		8	7		
2. I think homeworks for which I have to prepare questions for a fellow student help me understand the topic well.	3	6	5		1
3. I prefer homeworks for which I have to prepare questions to traditional homework set from Textbooks	1	3	5	5	1

or Review Sheets.					
4. Homeworks for which I have to prepare questions result in the next lesson being more interesting.	5	7	2	1	
5. Homeworks for which I have to prepare questions suit my learning style	1		9	4	1

In addition students were asked to comment freely. These are the comments made by some of them.

Comments:

I prefer working through questions from books to reach an answer as I feel I can learn the method to answer the questions.

I find making up questions easier but I don't think it improves my knowledge of what I am doing. Questions from Review Sheets etc. help more when I go through them after and work out mistakes I make.

I think a variety of homeworks from textbooks and Review Sheets and setting our own questions is the best way for learning through homework.

Although often they don't actually work, I can often work out why, and change it.
They help you understand where you are going wrong and make you think into it and all the possible outcomes. (I liked this comment).

These homeworks help me understand the topic much better and provide good practice but I think I learn more from Review Sheets.

I think questions for other people are good but I like doing traditional questions much more to work it out for myself and then do a question for others after that.

If I had a choice I wouldn't.

Some of these comments were encouraging. I felt that it was worthwhile continuing this method, refining it as I used it more and more. The students, quite sensibly indicated that they liked this method as long as it was mixed with other methods too. So variety seemed to be what they liked. I was pleased that their comments seemed to be well-balanced and thought through with some care.

I was keen to see how students would manage question setting in a Pure Maths module. I suspected that they would find it easier and that they would come up with questions that were more precisely expressed and that their questions would give rise to unique solutions.

The first Pure Maths module that I taught them was Further Pure 1. This module involves more algebraic topics and the students were on slightly more familiar ground.

Some tasks set are given below with more detail in the appendix.

Task 3: Roots of quadratics

Task 4: General Solutions of Trigonometric Equations – 26th April 2006
These questions evoked some discussion on problems such as how to use the language of mathematics appropriately and why a problem may not have a real solution. These are both important issues in mathematics and such discussions naturally arise throughout any mathematics course. However, there was a sense of discovery as they, themselves, found the obstacle to a real solution, rather than having it contrived via the textbook or the teacher.

Another interesting observation was that students could include graphs to support their question. One student did this and this gave emphasis to the visual aspect of learning.

The discussions I observed suggested that students were reflecting on the problems and were doing so in a more sophisticated than at the beginning of the course.

The students were approaching the half-term break before study leave for the summer exams. I felt this was a good time to canvas their views again. I did this via a second questionnaire which I used specifically for this topic.

QUESTIONNAIRE 2

	Strongly agree	Agree	Disagree	Strongly disagree
1. I am confident about finding the general solution of trig equations.	4	6		
2. I think that setting a question for a fellow student helped me consolidate my understanding.	1	8		1
3. Preparing a question for a fellow student helped me understand how to use the language of maths for trig equations.		9	1	
4. I think that doing a fellow student's prepared question improved my understanding.	1	6	3	
5. I think that sharing questions with fellow students is an enjoyable way of learning.	1	8		1

Generally the students indicated they found this exercise useful. One student consistently indicated a dislike. He nevertheless always participated fully in the exercise and did so with good cheer.

Task 5: Matrices – 9th May 2006

As we approached the end of the AS course we inevitably started revision. I decided that we would do the matrices revision via problem sharing. Students were asked to revise the topic for homework and then prepare an exam style question for sharing. This went particularly well, maybe because it was a topic with which students were more familiar. Good questions were set requiring manipulation of matrices, diagrams of their effect on a set of points in 2-D space and interpretation of the transformations they represented. I could see that students were becoming used to setting questions with multi-stage solutions which drew on different methods of reasoning. A particularly interesting discussion arose out of one question in which a student slipped the zero matrix into the product of 5 matrices. When his student partner came to interpreting the product, she said "it does nothing". The discussion that ensued involved the meaning of "zero" and "nothing" in a mathematical context. I took the opportunity to point out the different zeros used in mathematics – for example: the real number zero, the complex number zero, the zero vector, the zero function……. I have done this before, but the discussion held on this occasion seemed to grab their attention more since its source was one of their problems specifically.

Modifications

Since my students were not overwhelmingly enthusiastic about setting problems **for homework** I decided to make a slight adjustment to the method. This was simply to get students to set problems for one another as soon as we had been through a new concept. I had seen this done very effectively in two lessons that I observed during our Mathematics and Science Faculty lesson observation cycle. Both teachers used the method in their lesson and had clearly done it often before because the exercise was executed in a very natural way by the students. Under these circumstances the exercise seemed less pressured since the problem solving requirements were more specific, less open ended.

Modified Task: Composition of functions
Set a question involving two different functions. Decide what compositions of these functions you will get your student pair to perform. You should require 2 or 3. Solve your problem yourself so that you can check your solutions together.
Students found this a natural activity to complete. It did not take long and was completed with success by all pairs. However, I observed that the tasks were undertaken in a more perfunctory way.

CONCLUSIONS

- Decision mathematics problems were easiest to set and students found them enjoyable.
- Pure mathematics problem setting and solving had varying success and enjoyment, but led to opportunities to discuss the language of mathematics.
- Statistics problems were more difficult and not as appealing. I did not include an example of this as it did not go particularly well. (I will need to improve here)
- Setting and solving problems improved as the year progressed.
- During these activities students were animated and enjoyed them for the most part.
- Even though students were setting the problems themselves, it was important to be clear about the context and expectations. Where the exercise was too loosely framed, there was least success.
- Both questionnaires indicate that students felt positive about the activities and felt that they helped to consolidate their understanding.
- Observations of the students during these activities showed students fully engaged. None was passive and almost all were fairly animated at least even though a few students said they did not particularly like these activities.
- The majority of students suggested their understanding of the topics had been enhanced through the activity to an extent
- Opportunities arose during the activities to discuss the use of the language of mathematics. Although it is difficult to measure the success of this, as their teacher, I felt there was more focus on this than might have arisen in other classroom situations.

I am determined that I will continue to incorporate student question setting in lessons; sometimes set for homework; sometimes set in the lesson. I am content that it is yet another method I can use to bring variety to my lessons while improving, I hope, my students' participation and, ultimately, achievement. I hope that I may refine this method as I learn how to use it more and more effectively.

My Own Reflections
I am not a risk taker and prefer to work with methods with which I feel comfortable. This cannot be best for my students. I know that I need to serve their learning needs. What has helped me to feel confident about doing this research – which I felt I had to do for the benefit of my students – was the support of my students and my colleagues.

I don't think my colleagues are fully aware of the impact that **their** teaching and sharing of ideas has had on me. Without knowingly doing so, they have reinforced my confidence in what I have been doing because they employ similar methods and enthusiastically discuss successful lessons. These daily accounts (and occasional lesson observations) of classroom activities has been enormously enriching and encouraging and I am grateful to be working in such a dynamic teaching and learning environment.

My students, too, have been great. Each one entered the research with their own personal imprint, ranging from very enthusiastic and involved to not quite so keen and preferring the more formal, structured teacher-led approach. The message thundered over was that I dare not slip into old ways and must vary my methods and seek new approaches.

Of one thing I am absolutely certain is that I still have much to do to refine my teaching. Doing this research has helped me to feel less anxious about taking what I had perceived were risks. Other research is there to support me, as are my students and colleagues.

HOW I MIGHT FOLLOW UP OR EXTEND MY RESEARCH

I am mindful that I could have given my students the chance to be more evaluative about this method and encouraged them to describe their thinking, planning and learning. There is evidence that this aspect of "metacognition" is of benefit to students. Schoenfeld, A. H. (1987) I am sure it will need time, but it will give me something new and fresh to try as I have seldom explicitly done this before.

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APPENDICES:

1. THE TASKS IN MORE DETAIL (with some repetition)

Task 1: Algorithms – 13th September 2005

Students were set the task to make up an algorithm. The idea was that the algorithm should result in an outcome which would not initially be known to the student who was to apply it. This idea was "borrowed" from a colleague who had tried it out with his own students when teaching them Decision 1 and found it highly workable.

The task was set a day in advance of the lesson. Algorithms that the students came up with were:

- Making a paper aeroplane. This was successfully made by the student pair.
- Drawing a simple but abstract shape. Successfully done with pencil.
- An algorithm that calculates the longest chain of even numbers in a given series of integer numbers – probably off the internet. A good challenge.
- Directions to a student's home.
- Finding out whether a number is prime.
- A sorting algorithm.
- Making a cup of tea. Obvious at the outset.

Comments:

The first six were largely successful because of the element of surprise – not knowing what the algorithm would do until a few steps has been tried. Making a cup of tea was obvious from the start. The student afterwards realized that he should have devised an algorithm which was such that they would only know the outcome once a pass of the algorithm had been completed.

Students learned that it was important to have an algorithm in which the steps were clear and unambiguous. They could see this in their own algorithms when they tried it out on their student pair and this was reinforced when they tried their student pair's algorithm.

During the activity students were very animated and seemed to enjoy doing each other's questions. Three students rose to the challenge of finding unusual and challenging algorithms.

Task 2: Graph theory and networks – 20th September 2005

13 out of 17 students attempted this task.

Students were set the task to make up a question about graphs and networks. They had just spent the lesson researching graphs and network via a verbal and visual group exercise and the aim of this activity was to consolidate their learning and to differentiate between deeper and superficial understanding. Students were told they would swap their questions with another student, at random, the next day.

These were some of the questions set:

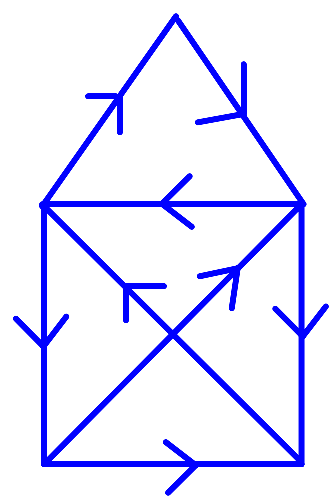
1. Produce a complete graph which has 4 vertices. Additionally each vertex has a loop.
Comments:
The graph was well defined with only one graph resulting (up to isomorphism) Student drew the correct graph. Some discussion resulted because one student thought that the definition of the graph needed to include specifying the number of edges.

Teacher input: Because the graph was defined as complete, the number of edges did not, in fact, need to be specified. The loops were added afterwards.
This was a clearly, simply stated problem which was appropriate for this level.

2. The graph is a connected graph which contains a tree, and a cycle joining 3 vertices. The graph contains no loops, multiple edges and cannot be made into one full cycle. All of the edges are of equal lengths.
Comments:

The definition of this graph contained contradictory information which confused the student who had to draw it. This generated a lot of discussion about connected graphs, trees and cycles and the fact that it is not necessary to specify that the edges are of equal lengths. However, the student wanted to make an interesting graph. This gave him and his paired student something to think about and after the discussion they realized that it is important to give clear, unambiguous information in problem setting. This was shared with the class too.

3. Draw four vertices in the shape of a square. Label the vertices A, B, C, D.
Ensure the graph is a Hamiltonian cycle, making sure no edges cross.
Now complete the graph.
Make a fifth vertex, so that they form a pentagon shape.
Join the fifth vertex to the nearest two vertices
Now turn it into a digraph, also ensure it's Semi-Eulerian, showing where you started. This is the diagram she intended.



4. **Comments:**
This student tried to put lots of ideas into her problem mixing the notion of an algorithm with graph theory. She and her student pair needed to discuss what she intended the resulting graph to look like. Some of the steps were not necessary to produce the required graph. The notion of an isomorphism could have been stressed beforehand, but it was probably useful to do so afterwards. After their discussion they appreciated that the correct use of maths language and objects was not as simple as appeared at first.

5. (1) Draw a tree which has 8 vertices and 7 edges. The vertices are labelled A, B, C, D, E, F, G and H. B, D, E and F make a vertical straight line.
(edges) BA, BD, BC, DE, EF, EG, EH.
(2) 5 vertices: M,N,O,P,Q; edges MN, MO, MP, MQ, OO, QQ.
Comments:
(1) This problem went well with isomorphic diagrams.
(2) This was rather trivially defined but yielded isomorphic diagrams.

Task 3: Roots of quadratics

The first topic was on roots of quadratics – the connection between the roots, α and β and the coefficients a, b and c of the quadratic equation. $ax^2 + bx + c = 0$. Specifically,

$\alpha + \beta = \dfrac{-b}{a}$ and $\alpha\beta = \dfrac{c}{a}$.

Students were set the homework task of setting a problem involving a quadratic equation of the type suitable for this topic. No further detail was given as I felt that it was fairly clear what kind of question to set.

These are some of the problems set.

- The quadratic $7x^2+3x+11=0$ has roots α and β. Find a quadratic with roots

  $6\alpha+\dfrac{2}{\beta}$ and $6\beta+\dfrac{2}{\alpha}$ (J)

Comments:
This looks like a standard question which is sufficiently difficult – just the kind a student might expect in an exam. However, the fractions involved were rather nasty and the student's pair struggled to work with the fractions. The student should have worked through the problem carefully beforehand – as expected as part of her homework preparation – so that she could see that was unnecessarily complicated and not enhancing the knowledge and methods associated with roots of a quadratic.

- If $5x^2+101x+5=0$ has roots α and β, write a new equation so that its roots are

  $2(\alpha+\beta)$ and $3\alpha\beta$ (H's question given to N)

Comments:
A straightforward question which worked well and N managed to solve it well with using the right algebraic approach.

- The equation $x^2-5x-7=0$ has roots α and β. Find the quadratic equation whose roots are α^3 and β^3 (N's question given to H).

Comments:
Quite a tough question because it needs the expansion of $(\alpha+\beta)^3$. H consulted the textbook to get the expansion thereby improving her knowledge on this topic. She arrived at the correct quadratic

- Write down the quadratic equation whose roots have the sum -9 and product 6.
  The roots of this equation are α and β. Find the quadratic equation whose roots are α^2 and β^2. (N's question given to T).

Comments:
This problem was clearly and unambiguously stated and T found it easy to solve.

- T's problem for N was very briefly stated and needed to have details given verbally.

  " $x^2+4x+12$ find $\dfrac{1}{\alpha^2}$ and $\dfrac{1}{\beta^2}$ ".

Comments, outcome and teacher input:
Once the problem was better posed N solved it successfully. (Roots are complex, but this doesn't matter)

Task 4: General Solutions of Trigonometric Equations – 26th April 2006
Students were given the task to set a trig equation for which their student pair was to find the general solution. I have included two examples. The first one was set by a student with a strong verbal learning style who likes to read maths textbooks. He set this in the spirit of making the exercise fun by adding a touch of whimsy. I have not changed his grammar.

- "Norman the rivet collector has two great loves: Rivet collecting, and general solutions to trigonometric equations. Or at least for this week and this question these are his two loves. However poor Norman has hit a slight problem – he doesn't know any trigonometry. In his reckless youth he preferred "youth culture" and "friends", though now he knows much better. Fortunately his favourite mathematician rivet Joe is an expert at trigonometry, and so could answer Norman's poor problem, but the question is, can you?

Norman's problem was:
What is the general solution, in degrees (Norman doesn't believe in none of these new fangled radians. He's an old fashioned fellow) to:
$\sin 4\theta + 25 = 1$
Why he wants to know, nobody knows, but it matters to poor Norman – and would you want to see a sad rivet collector"

The first point his student pair noticed was that this equation did not have a (real) solution. A discussion ensued about this and the need to use the language of mathematics correctly to avoid ambiguity. The equation was then given as $\sin\left(4\theta + 25\right) = 1$. Including the brackets made this possible, and easy, to solve.

A more orthodox question set was:

"Find the general solution for the equation $\sin 3\theta = \frac{\sqrt{2}}{2}$, in radians.

Find all possible values for θ, when $-4\pi \le \theta \le 4\pi$
Make sure you include a graph in your working".

A student with a strong visual learning style set this question, hence the inclusion of the comment about including a graph in the working.

Students were invited to give feedback to the class on the use the use of mathematical language in the problems. These are some of their comments.

Don't forget brackets - eg $\sin 2\theta + 25 = 1$ -**T**

Remember special triangles-**J**

Awkward numbers used - don't be afraid of decimals - **S**

Be careful with signs **- L**

Complicated question set **A** - hmmmmmm

Be careful about the period of function.

Students were clearly reflecting on the problems and this was more sophisticated than it had been at the beginning of the course.

Task 5: Matrices – 9th May 2006
This went particularly well, maybe because it was a topic with which students were more familiar. Good questions were set requiring manipulation of matrices, diagrams of their effect on a set of points in 2-D space and interpretation of the transformations they represented. One student criticised her student partner for writing

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2\frac{1}{2} \\ \frac{5\sqrt{3}}{2} \end{bmatrix}$$

This is a common abuse of the "equals" sign. It was encouraging to see that students were thinking about how they should use the language of mathematics correctly.

An interesting discussion arose out of a problem set which involved a sequence of five transformations represented by a product of 5 matrices. The student slipped the zero matrix into the product. When his student partner came to interpreting the product, she said "it does nothing". The discussion that ensued involved the meaning of "zero" and "nothing" in a mathematical context. I took the opportunity to point out the different zeros used in mathematics – for example: the real number zero, the complex number zero, the zero vector, the zero function……. I have done this before, but the discussion held on this occasion seemed to grab their attention more since its source was one of their problems specifically.

2. ISSUES AND SUCCESSES

Task 1: Algorithms
Just over half the class produced good algorithms. Some were uncertain how to do the activity or did not do it at all for homework. Some brought in rather scrappy work. I realized I had to be more precise in my instructions if all students were to benefit fully.

However, I was particularly pleased that three students found interesting algorithms and did the exercise in the spirit of a challenge.

Task 2: Graph theory
I had introduced this topic in an investigative way in which the students used verbal and visual activities to learn the definitions. A few of their questions were not well structured and used a number of definitions that were not consistent. This made it a bit frustrating for the student pair to solve. Also, there were several other ideas which were rather more complicated than students would be expected to deal with in exams. This exercise highlighted the importance of being very precise in posing problems and using new language and ideas correctly and unambiguously.

However, the positive side of this was that the students discussed the problems and this, I believe, was the point at which they became confident at challenging one another's questions and being critical of them.

Task 3: Roots of quadratics
A few students were rather ambitious, producing questions that ended up some rather nasty algebra. I pointed out that the object of the exercise was not to do complicated algebra (necessarily), but to understand the relationship between coefficients of a quadratic and its roots.

However, the exercise highlighted the need to focus on the essence of the topic and for most students it lent itself well to this style of questioning.

Task 4: General solutions of trigonometric equations
There were no issues other than that some students used the language of mathematics in a cavalier way.

However, I regarded this as a good opportunity to focus on this problem via a brief discussion.

Task 5: Matrices
The main issue was, again, with the use of the language of maths. The discussion that resulted addressed this.

On the positive side, it was evident to me that my students were becoming better at question setting and that they did this better with a topic with which they were familiar, as it was a revision exercise.

Finally, doing an impromptu question setting exercise in class on compositions of functions went very well and felt very natural for me and for the students.

3. COMMENTS ON STUDENTS' QUESTION SETTING STYLES

The Verbal Student: **T** always grasped the maths required. He liked to set questions that were predominantly verbal and not typical of the style of question in most mathematics textbooks.

The Very Able Student**: N (L and A_1)** – a very strong mathematician – set orthodox, but challenging, questions that usually had more than one part to it and demanded a deeper understanding of the underlying mathematics.

The Risk Taker**: A_2** had a particularly individual style, using current issues (e.g. war in Iraq) to set his questions in an interesting context – this, sometimes to the detriment of the problem set. He tried to be overambitious and did not always check that his problems were solvable'

The Conservative: A number of students went for "safe" options. They usually looked in a textbook and varied a problem from it slightly.

The Forgetful: Each time one or two of students "forgot" to do their preparation, and had to produce an impromptu question.